



QUANTITATIVE METHODS FOR MANAGERS

Monday 12th December 2022

Time allowed

Three hours

Instructions

- Ensure that you pay particular attention to words in **bold**.
- Write the question number next to each answer in your answer booklet.
- You are **not** required to rewrite the question in your answer booklet.
- Show all your workings.

Information

- Different questions may carry a different number of marks.
- Marks for each question are shown in [].
- Questions start on page 8.

Advice

- Read each question carefully before you start to answer it.
- Use the full time permitted and check all your answers.

Materials

- Notes or books are **not** permitted.
- Non-programmable calculators are permitted.



ICM

STATISTICAL FORMULAE

Frequency Distributions

Arithmetic Mean

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Weighted average

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

Range

Range = Highest value – Lowest value

Quartile deviation

$$\text{Quartile deviation} = \frac{\text{Upper quartile} - \text{Lower quartile}}{2}$$

Standard deviation

$$S.D. = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{\sum f_i}}$$

Variance

$$\text{Variance} = (S.D.)^2$$

Coefficient of variation

$$\text{Coefficient of variation} = \frac{\text{S.Deviation}}{\text{Mean}} \times 100$$

Pearson coefficient of skewness (Sk)

$$Sk = \frac{3(\text{Mean} - \text{Median})}{S.D.}$$

Standard Errors and Confidence Intervals

Standard normal deviation

$$z = \frac{x - \mu}{\sigma}$$

Standard error of the mean

$$S.E. = \frac{s}{\sqrt{n}}$$

Standard error of a proportion

$$S.E. = \sqrt{\frac{pq}{n}}$$

where p = sample proportion, and $q = 1 - p$.

Confidence Interval for population mean (μ)

$$\bar{X} \pm Z_{\alpha} \times S.E.$$

Confidence Interval for population proportion (π)

$$p \pm Z_{\alpha} \times S.E.$$

Regression/Correlation

Regression lines

Regression line of y on x

Line equation: $y = a + bx$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad a = \frac{\sum y - b\sum x}{n}$$

Correlation Coefficient (r)

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

Rank Correlation Coefficient

$$\text{Spearman's coefficient of rank correlation} = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where n = the number of pairs, and d = the difference between ranking of the same item in each series.

Index Numbers

Laspeyres price index

$$\text{Index} = \frac{\sum(p_n \cdot q_o)}{\sum(p_o \cdot q_o)} \times 100$$

Paasche price index

$$\text{Index} = \frac{\sum(p_n \cdot q_n)}{\sum(p_o \cdot q_n)} \times 100$$

Price relative

$$\text{Price relative} = \frac{P_n}{P_o} \times 100$$

Base changing

$$\text{New index number} = \frac{\text{Old index number}}{\text{Old index number of new base period}} \times 100$$

Asset revaluation

$$\text{New valuation} = \text{Original value} \times \frac{\text{New price index}}{\text{Original price index}}$$

ANSWER ANY FIVE QUESTIONS FROM THE FOLLOWING EIGHT QUESTIONS

1. Table 1 shows the results of an experiment that has been conducted on an engine measuring the temperature of the exhaust and the oil consumption.

Table 1: Experiment results

Exhaust Temp (C)	160	160	170	170	180	190	200	220	240
Oil Consumption (ml)	276	305	309	302	259	334	302	371	341

- (a) Plot the scatter diagram of the data provided in Table 1. [2 marks]
- (b) Calculate the line of best fit using the least squares method. [8 marks]
- (c) Calculate the predicted oil consumption at an exhaust temperature of 260°C. [2 marks]
- (d) Calculate the coefficient of determination (R^2). [6 marks]
- (e) Outline the relationship between exhaust temperature and oil consumption. [2 marks]
2. The management of a coal fired power station is studying the plants operational setup in order to comply with the latest emissions standards.

Coal fired power station setup

Maximum sulphur emissions are 3,000 PPM and maximum particulate smoke emissions are 12 Kg/hour. Coal is brought to the plant where it is crushed and placed into the combustion chamber. The conveyor loading system has a capacity of 20,000 Kg/hour regardless of coal type. Two types of coal are used:

- Type A is a hard and clean burning coal with low sulphur content but relatively expensive.
- Type B is a clean, soft and smoky coal with a high sulphur content.

Table 2: Coal data

Coal Type	Sulphur emissions	Particulate emissions per 1,000 Kg	Thermal Output per 1,000 Kg	Crusher Volume 100 Kg/hr
A	1,800 PPM	0.5Kg	24,000	16
B	3,800 PPM	1.0Kg	20,000	24

The management wants to know the maximum electric power output from the plant in relation to the emissions standards and the plant operating limitations.

- (a) Prepare this problem as a linear programme stating the objective equation using the data provided in Table 2. [8 marks]
- (b) Draw a graphical estimate of the solution to question 2(a). [6 marks]
- (c) Calculate the optimal values of the coal types including the maximum thermal output. [6 marks]

3. (a) Explain what is meant by the terms 'simple interest' and 'compound interest'. Use examples to support your explanation.

[4 marks]

(b)

Table 3: Project data

Time period	Project A	Project B
0	-18,000,000	-6,000,000
1	0	2
2	0	2,000,000
3	5,000,000	2,000,000
4	6,000,000	2,000,000
5	7,000,000	2,000,000
6	8,000,000	0
7	9,000,000	0
8	10,000,000	0

Calculate each of the following using the data in Table 3:

- (i) The payback period for Project A and Project B
- (ii) The average rate of return for Project A and Project B
- (iii) The net present value (NPV) for Project A and Project B using a discount rate of 10%.

[4 marks]

[4 marks]

[6 marks]

- (c) Outline the way the choice of capital appraisal method affects company investment decisions.

[2 marks]

4. Table 4 shows the sales of a car model over the last three years:

Table 4: Car sales

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2019	66,000	106,000	140,000	82,000
2020	73,000	119,000	165,000	91,000
2021	85,000	130,000	205,000	100,000

- (a) Calculate the values for the trend component using moving averages.
- (b) Plot the data provided in Table 4 and calculated trend values from question 4(a) on a scatter diagram.
- (c) Estimate the seasonal variation component.
- (d) Calculate the forecast future sales for each of the following, considering both seasonal variation and trend:
- (i) Quarter 1 2024
- (ii) Quarter 3 2024

[4 marks]

[4 marks]

[8 marks]

[2 marks]

[2 marks]

5. (a) Define what is meant by the term independent event. [2 marks]
- (b) (i) Two standard dice are thrown together.
Calculate the probability of scoring six on each die. [2 marks]
- (ii) Suggest whether the resulting score is a dependent or independent event, including a reason for your suggestion. [2 marks]
- (c) Define what is meant by the term mutually exclusive event. Use an example to support your definition. [4 marks]
- (d) A dice (die) is claimed to be fair. It is rolled 120 times and the results are shown in Table 5:

Table 5: Rolling results

Number thrown	Observed frequency
1	12
2	22
3	18
4	28
5	26
6	14

- (i) Prepare the expected frequency table. [6 marks]
- (ii) Explain the way to establish whether the dice (die) is fair or not. [4 marks]
6. (a) Explain what is meant by each of the following terms:
- (i) Null hypothesis [2 marks]
- (ii) Alternative hypothesis [2 marks]
- (b) Explain the **two** types of error to avoid in hypothesis testing. Use an example to support your explanation. [4 marks]
- (c) A wire drawing machine produces wire with a tensile strength with a mean of 100Kg and a standard deviation of 14Kg. Customers have asked for stronger wire and a new technique has been introduced into production. A sample of 49 components has been tested, giving a mean of 104Kg.
- Calculate whether the new technique has produced wire with a higher tensile strength, testing at the 5% significance level. [8 marks]
- (d) Explain the circumstances in which it is appropriate to use the 'student t distribution' in calculations. [4 marks]

7. Table 6 shows the weekly earnings of 30 employees at an engine test facility:

Table 6: Employee earnings

Quantity	Job	Weekly earnings
3	Rig Engineers	£180
1	Admin assistant	£130
3	Maintenance technicians	£170
20	Test Technicians	£250
2	Senior Managers	£500
1	Director	£1,000

- (a) Draw a histogram to display the data provided in Table 6. [6 marks]
- (b) Calculate each of the following for the weekly earnings:
- (i) The arithmetic mean [4 marks]
 - (ii) The modal value [3 marks]
 - (iii) The median value [3 marks]
 - (iv) The standard deviation [4 marks]
8. It is estimated that if a price of £15 is charged for a product, then five units per period will be sold. If the price is lowered to £13, it is estimated that six units will be sold per period. The supply curve for the product is given by the formula:

$$P=3Q+5$$

- (a) Calculate the demand equation, with the assumption the demand relationship is linear. [10 marks]
- (b) Calculate the equilibrium price and quantity (the price where quantity demanded = quantity supplied). [10 marks]

END OF QUESTIONS