



QUANTITATIVE METHODS FOR MANAGERS

Monday 12th September 2022

Time allowed

Three hours

Instructions

- Ensure that you pay particular attention to words in **bold**.
- Write the question number next to each answer in your answer booklet.
- You are **not** required to rewrite the question in your answer booklet.
- Show all your workings.

Information

- Different questions may carry a different number of marks.
- Marks for each question are shown in [].
- Questions start on page 8.

Advice

- Read each question carefully before you start to answer it.
- Use the full time permitted and check all your answers.

Materials

- Notes or books are **not** permitted.
- Non-programmable calculators are permitted.

STATISTICAL FORMULAE

Frequency Distributions

Arithmetic Mean

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Weighted average

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

Range

Range = Highest value – Lowest value

Quartile deviation

$$\text{Quartile deviation} = \frac{\text{Upper quartile} - \text{Lower quartile}}{2}$$

Standard deviation

$$S.D. = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{\sum f_i}}$$

Variance

$$\text{Variance} = (S.D.)^2$$

Coefficient of variation

$$\text{Coefficient of variation} = \frac{\text{S.Deviation}}{\text{Mean}} \times 100$$

Pearson coefficient of skewness (Sk)

$$Sk = \frac{3(\text{Mean} - \text{Median})}{S.D.}$$

Standard Errors and Confidence Intervals

Standard normal deviation

$$z = \frac{x - \mu}{\sigma}$$

Standard error of the mean

$$S.E. = \frac{s}{\sqrt{n}}$$

Standard error of a proportion

$$S.E. = \sqrt{\frac{pq}{n}}$$

where p = sample proportion, and $q = 1 - p$.

Confidence Interval for population mean (μ)

$$\bar{X} \pm Z_{\alpha} \times S.E.$$

Confidence Interval for population proportion (π)

$$p \pm Z_{\alpha} \times S.E.$$

Regression lines

Regression line of y on x

Line equation: $y = a + bx$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad a = \frac{\sum y - b\sum x}{n}$$

Correlation Coefficient (r)

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

Rank Correlation Coefficient

$$\text{Spearman's coefficient of rank correlation} = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where n = the number of pairs, and d = the difference between ranking of the same item in each series.

Index Numbers

Laspeyres price index

$$\text{Index} = \frac{\sum(p_n \cdot q_o)}{\sum(p_o \cdot q_o)} \times 100$$

Paasche price index

$$\text{Index} = \frac{\sum(p_n \cdot q_n)}{\sum(p_o \cdot q_n)} \times 100$$

Price relative

$$\text{Price relative} = \frac{P_n}{P_o} \times 100$$

Base changing

$$\text{New index number} = \frac{\text{Old index number}}{\text{Old index number of new base period}} \times 100$$

Asset revaluation

$$\text{New valuation} = \text{Original value} \times \frac{\text{New price index}}{\text{Original price index}}$$

ANSWER ANY FIVE QUESTIONS FROM THE FOLLOWING EIGHT QUESTIONS

1. (a) Table 1 is a frequency table showing the quantity of grain in Kg used by a farmer over a period of 50 weeks:

Table 1

Kilograms (Kg)	Frequency
Less than 100	3
100 < 120	7
120 < 140	10
140 < 160	14
160 < 180	9
180 < 200	5
More than 200	2

- (i) Draw a histogram for this data and comment on its shape. [5 marks]
 (ii) Calculate the mean and standard deviation of grain. [10 marks]
 (b) Over a similar period in the previous year, the data showed a coefficient of variation of 25%.
 Describe the farmer's usage of grain over the two years. [5 marks]

2. A visitor attraction is investigating its charges during peak periods for a particular annual holiday period over the last three years. Table 2 shows the average number of visitors for the three daily sessions:

Table 2

Year	Daily Sessions	Average number of visitors
2019	1	1,200
	2	1,412
	3	1,810
2020	1	1,320
	2	1,520
	3	2,102
2021	1	1,530
	2	1,670
	3	2,310

- (a) Plot a graph of this data and comment on any trends. [4 marks]
 (b) Calculate an appropriate moving average to show any underlying trend. [8 marks]
 (c) Calculate any seasonal components, assuming that an additive model is the most appropriate way to describe this time series. [5 marks]
 (d) Forecast the average number of visitors to this visitor attraction in daily session 1 in 2022. [3 marks]

3. **Sickness Absence Case Study**

The hotel manager wants to estimate the mean number of sickness absence days per employee during the last year. To do this, the manager examined the records of a sample of 60 employees and found that the mean of the sample was 10.8 days, with a standard deviation of 3.2 days. Stress was one of the causes of 40 staff members' absence.

- (a) Calculate a 95% confidence interval estimate for the overall mean number of days absent through sickness per company employee. Give your answer correct to 2 decimal places. [6 marks]
- (b) Explain the number of employees required as a sample for the manager to be 99% confident that the sample mean differed from the true overall mean by a maximum of 0.5 days. [8 marks]
- (c) Calculate a 90% confidence interval estimate of the overall proportion of employees for whom stress was a cause of their sickness absence. Give your answer correct to 2 decimal places. [6 marks]

4. Table 3 shows the average annual mileage per vehicle and total repair and maintenance costs (£) of vehicles for a taxi company from Jan to Aug.

Table 3

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Average mileage	28,000	20,000	16,000	20,000	24,000	20,000	14,000	16,000	30,000
Total costs (£)	44,000	28,000	24,000	36,000	44,000	40,000	34,000	32,000	

- (a) Draw a scatter diagram for the above data. [4 marks]
- (b) Calculate the appropriate least squares regression line so that total costs (Y) can be predicted from the average annual mileage (X) of the vehicles. [14 marks]
- (c) Calculate the predicted repair and maintenance costs for September. [2 marks]

5. A food wholesaler has analysed the monthly amount spent by its customers and it has found that it is normally distributed with a mean of £6,200 with a standard deviation of £1,500.

- (a) Estimate the percentage of customers that will have a monthly spend of:
- (i) Over £7,500 [5 marks]
 - (ii) Less than £5,500 [5 marks]
 - (iii) Between £5,500 and £7,500 [5 marks]
- (b) The wholesaler is going to reward the highest spending 5% of its customers with free gift vouchers.
Suggest the level of spend at which the wholesaler should offer this incentive, including reasons for your suggestion. [5 marks]

6. Table 4 shows the weekly demand for a particular brand of bottled water over a period of 50 weeks:

Table 4

57	53	56	75	46
47	43	54	43	51
62	41	73	65	45
72	50	59	68	56
71	52	58	79	60
77	38	54	59	65
38	54	80	54	56
57	55	67	67	82
70	61	83	56	42
49	56	53	68	52

- (a) Write a cumulative frequency distribution showing the weekly demand for the bottled water using the class intervals 30-40, 40-50, 50-60, etc. [5 marks]
- (b) Draw a labelled ogive (cumulative frequency) graph. [5 marks]
- (c) Using the ogive (cumulative frequency) graph drawn in question 6 part (a):
- (i) Estimate the median and quartile deviation of the weekly demand [5 marks]
- (ii) Estimate the proportion of time that the weekly demand falls between 65 and 80 bottles of water [5 marks]

7. Table 5 shows survey data on annual sales growth as a percentage of the previous year's growth during the last three years for a number of small companies:

Table 5: Sales Growth (%)

Region	No. of companies	2018	2019	2020	2021
North	50	-1.0	2.5	5.3	6.3
South	85	3.2	2.4	4.2	4.9
East	70	7.6	0.6	2.5	4.6
West	35	5.5	-0.7	-2.4	1.0
Central	60	1.7	2.3	7.3	6.8

Plot the charts using the information in Table 5:

- (a) A pie chart showing the percentage of companies in each region [8 marks]
- (b) A line graph comparing the average sales growth over the three years [5 marks]
- (c) A multiple bar chart comparing the sales growth for the regions over the three years [7 marks]

8. Table 6 shows the number of defective components that are being produced in five electronic goods factories within the same seven day time period. The owner of the factories wants to test the hypothesis that there is no relationship between the quality of work and the factories producing that work.

Table 6

Electronic Goods Factory	A	B	C	D	E
Number of defective components	56	33	27	31	53

Use the Chi-Square test to test this hypothesis at the 5% level of significance.

[20 marks]

END OF QUESTIONS